8.6: PROBLEM DEFINITION

Situation:

Liquid is draining out of a tank through an orifice at the bottom.

Find:

The $\pi-$groups in the form.

$$\frac{\Delta h}{d} = f(\pi_1, \pi_2, \pi_3)$$

.

PLAN

The fluid density and specific weight must also be included so functional form is

$$\Delta h = f(d, D, \gamma, t, h_i, \rho)$$

Use the step-by-step method.

SOLUTION

Δh	L	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0	$\frac{\Delta h}{d}$	0
t	T	t	T	t	T		
ρ	$\frac{M}{L^3}$	ρd^3	M				
D	L	$\frac{D}{d}$	0	$\frac{D}{d}$	0	$\frac{D}{d}$	0
d	L						
γ	$\frac{M}{L^2T^2}$	γd^2	$\frac{M}{T^2}$	$\frac{\gamma}{\rho d}$	$\frac{1}{T^2}$	$\frac{\gamma t^2}{\rho d}$	0
h_1	L	$\frac{h_1}{d}$	0	$\frac{h_1}{d}$	0	$\frac{h_1}{d}$	0

In the first step, length is taken out with d. In the second step, mass is taken out with ρd^3. In the third step, time is taken out with t. The functional relationship is

$$\boxed{\frac{\Delta h}{d} = f(\frac{D}{d}, \frac{\gamma t^2}{\rho d}, \frac{h_1}{d})}$$

This can also be written as

$$\frac{\Delta h}{d} = f(\frac{d}{D}, \frac{gt^2}{d}, \frac{h_1}{d})$$

8.24: PROBLEM DEFINITION

Situation:

Drag of a square plate placed normal to a free stream velocity.

Find:

The $\pi-$groups in the form

$$\frac{F_D}{\rho V^2 B^2} = f(\pi_1, \pi_2, \pi_3).$$

.

PLAN

The functional form of the dimensional equation is

$$F_D = f(\rho, V, B, \mu, u_{rms}, L_x)$$

There are 7 dimensional parameters, so there are 4 π-groups. Use the step-by-step method.

SOLUTION

Setting up the table,

F_D	$\frac{ML}{T^2}$	$\frac{F_D}{B}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho B^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 B^2}$	0
V	$\frac{L}{T}$	$\frac{V}{B}$	$\frac{1}{T}$	$\frac{V}{B}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρB^3	M				
B	L						
μ	$\frac{M}{LT}$	μB	$\frac{M}{T}$	$\frac{\mu}{\rho B^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho V B}$	0
u_{rms}	$\frac{L}{T}$	$\frac{u_{rms}}{B}$	$\frac{1}{T}$	$\frac{u_{rms}}{B}$	$\frac{1}{T}$	$\frac{u_{rms}}{V}$	0
L_x	L	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0

Length is removed in first step with B, mass is removed in second with ρB^3 and time is removed in the third with V/B. The function form is

$$\boxed{\frac{F_D}{\rho V^2 B^2} = f\left(\frac{\mu}{\rho V B}, \frac{u_{rms}}{V}, \frac{L_x}{B}\right)}$$

Other forms are possible. The π-group u_{rms}/V is referred to as "turbulence intensity."

8.36: PROBLEM DEFINITION

Situation:

A large venturi meter is calibrated with a scale model using a prototype liquid.

$\frac{1}{10}$ scale model, $p = 400\,\text{kPa}$.

Find:

The discharge ratio (Q_m/Q_p)

Pressure difference (Δp_p) expected for the prototype.

Assumptions:

Same fluids used; densities and viscosities are the same in model and prototype.

SOLUTION

Match Reynolds number

$$\begin{aligned}
\text{Re}_m &= \text{Re}_p \\
\frac{V_m L_m}{\nu_m} &= \frac{V_p L_p}{\nu_p} \\
\frac{V_m}{V_p} &= \left(\frac{L_p}{L_m}\right)\left(\frac{\nu_m}{\nu_p}\right) = \left(\frac{L_p}{L_m}\right)(1)
\end{aligned} \tag{1}$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$:

$$\begin{aligned}
\frac{V_m A_m}{V_p A_p} &= \left(\frac{L_p}{L_m}\right) \times (1) \times \left(\frac{L_m}{L_p}\right)^2 \\
\frac{Q_m}{Q_p} &= \left(\frac{L_m}{L_p}\right) \\
& \boxed{\frac{Q_m}{Q_p} = \frac{1}{10}} \\
C_{p_m} &= C_{p_p} \\
\left(\frac{\Delta p}{\rho V^2}\right)_m &= \left(\frac{\Delta p}{\rho V^2}\right)_p \\
\Delta p_p &= \Delta p_m \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2 ; \text{ where } \left(\frac{\rho_p}{\rho_m}\right) = 1^* \\
&= \Delta p_m (1) \left(\frac{L_m}{L_p}\right)^2 \\
&= 400\,\text{kPa} \times \left(\frac{1}{10}\right)^2 \\
& \boxed{\Delta p_p = 4.0 \text{ kPa}}
\end{aligned}$$

* because the same fluid is assumed to be used in both model and prototype

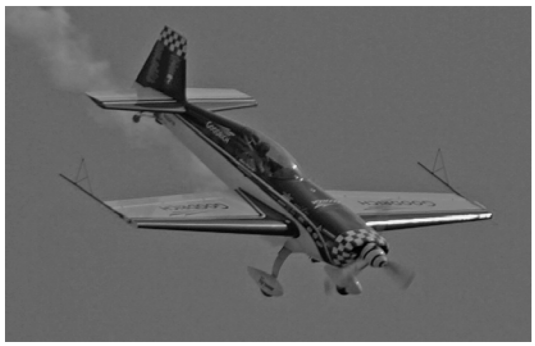
8.39: PROBLEM DEFINITION

Situation:

Lift force for an airplane.
The $\pi-$group for lift coefficient is

$$C_L = 2\frac{F_L}{\rho V^2 S}$$

Air: $\rho = 1.1\,\mathrm{kg/m^3}$, $C_L = 0.4$.
$V = 80\,\mathrm{m/s}$, $S = 15\,\mathrm{m^2}$.



Find:

Lift force (N).

PLAN

Use the specified value of $C_L = 0.4$ along with the definition of this π-group.

SOLUTION

From the definition of C_L:

$$\begin{aligned} F_L &= C_L\left(\frac{\rho V^2}{2}\right)S \\ &= 0.4\left(\frac{1.1\,\mathrm{kg/m^3}\times(80\,\mathrm{m/s})^2}{2}\right)\left(15\,\mathrm{m^2}\right) \\ &= 21,100\,\mathrm{N} \\ & \boxed{F_L = 21.1\,\mathrm{kN}} \end{aligned}$$